Accelerated Motion Equation Derivations

$$v_{avg} = \frac{\Delta x}{\Delta t} \quad , \quad v_{avg} = \frac{v_i + v_f}{2}$$

$$\frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}$$

$$\boxed{\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t} \qquad \text{(equation 4)}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

$$a\Delta t = v_f - v_i$$

$$\boxed{v_f = v_i + a\Delta t} \qquad \text{(equation 1)}$$

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t$$

$$\Delta x = \tfrac{1}{2}(v_i + v_i + a\Delta t)\Delta t$$

$$\Delta x = \tfrac{1}{2}\left[2v_i\Delta t + a(\Delta t)^2\right]$$

$$\boxed{\Delta x = v_i\Delta t + \tfrac{1}{2}a(\Delta t)^2} \qquad \text{(equation 2)}$$

Equation 4 $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$

$2\Delta x = (v_i + v_f)\Delta t$

$\Delta t = \frac{2\Delta x}{v_i + v_f}$

Equation 1 $v_f = v_i + a\Delta t$

$v_f = v_i + a\left(\frac{2\Delta x}{v_i + v_f}\right)$

$v_f - v_i = a\left(\frac{2\Delta x}{v_i + v_f}\right)$

$(v_f - v_i)(v_f + v_i) = 2a\Delta x$

$v_f^2 - v_i^2 = 2a\Delta x$

$\boxed{v_f^2 = v_i^2 + 2a\Delta x}$ (equation 3)

Eq 4 $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$

Eq 1 $v_f = v_i + a\Delta t$

$v_i = v_f - a\Delta t$

$\Delta x = \frac{1}{2}\left[(v_f - a\Delta t + v_f)\Delta t\right]$

$= \frac{1}{2}\left[(2v_f - a\Delta t)\Delta t\right]$

$\Delta x = \frac{1}{2}\left[2v_f\Delta t - a(\Delta t)^2\right]$

$\boxed{\Delta x = v_f\Delta t - \frac{1}{2}a(\Delta t)^2}$ (equation 5)