Vertical Circular Motion – Nonuniform Circular Motion

When an object moves in a circle whose plane is parallel to the direction of gravity (perpendicular to the ground), the speed of the object is not the same at all times and locations. This type motion is nonuniform circular motion. A good example of this type motion is a roller coaster cart traveling through a loop. The cart slows as it moves to the top of the loop and speeds up as the cart comes down the loop. Beware! A Ferris wheel is NOT an example of nonuniform circular motion because the wheel is driven by a motor so the speed will be the same at all times.

When we look at vertical circular motion, we generally will look at situations where to object undergoing the circular motion will only have two forces acting on it – gravity and one external force such as the normal force from a surface or the tension of a string.

Since the object is moving in a circle, the equations for centripetal acceleration and centripetal force still apply. In uniform circular motion, the NET force on the object was the centripetal force and this force had the same value at all points around the circle. In nonuniform circular motion, the NET force on the object is the centripetal force at only two points and the centripetal force will not have the same value at each point. These two points are the points that are of greatest interest for us. The two points are: the lowest point of the circular path and the highest point.

**For an object at the lowest point of the circular path (bottom of the circle):**

The object has two forces acting on it: gravity and some radial force from a surface or a string. The free-body diagram looks like this: The NET force on the object is:

 or 

W

radial force, Fr

At this point, Fc = NET force = 

The radial force, FN or Tstring , must be larger in

magnitude than the weight.

**For an object at the highest point of the circular path (top of the circle):**

The object has two forces acting on it: gravity and some radial force from a surface or a string. The free-body diagram looks like this: **NOTE – I have assigned positive direction as the direction from the object toward the center of the circle!**

Case 1 Case 2 Case 1 is far more common.

W

radial force, Fr

At this point, Fc = NET force = 

OR

W

radial force, Fr

 

You determine the direction of the radial force from the situation described in the problem. Case 1 is the case we usually see. Case 2 could be the situation of a car going over a hill – the radial force would be the normal force of the roadway on the car.

**Minimum speed of an object at the top of the circle**

In case 1, as the object moves through the top of the circle the centripetal force is provided by gravity (weight of the object) and the radial force. As the speed of the object decreases, the centripetal force necessary to keep the object in its circular path also decreases. Since the weight stays the same, the decrease in centripetal force must come from the decrease in the radial force. There is a speed at which the radial force drops to zero and the centripetal force is provided by the gravitational force alone. If the speed drops below this value, the object will not continue along its circular path. The minimum speed of an object at the top of a vertical circle is:

 where *r* is the radius of the circle in meters.